COMP 1805 B2

mICHAEL mAXWELL

101006277

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A1

1. Let x be the proposition “The data is secure”.

Let y be the proposition “The computer has a virus”.

* 1. The data is not secure and the computer has a virus.
  2. The data is secure if the computer does not have a virus.
  3. Either the computer has a virus or the data is secure.

1. Let a be the proposition “The economy is improving”.

Let b be “I make money”.

Let c be “I have a job”.

Let d be “I finish my degree at Carleton”.

* 1. If I finish my degree at Carleton and I make money then I make money.
  2. If I do not make money then I do not have a job.
  3. If I do not make money and I do not have a job then I did not finish my degree at Carleton.
  4. Let a be “I do exercise”.  
     Let b be “I am in shape”.  
     (a → b) ∧ (b → ¬a)
  5. Let a be “The sun is bright”.  
     Let b be “It is cloudy”.  
     a ∨ b
  6. Let a be “The road conditions are bad”.  
     Let b be “I am distracted”.  
     Let c be “I have an accident”.  
     (a ∨ b) → c
  7. Let a be “I can ski”.  
     Let b be “I can skate”.  
     Let c be “I like winter”.  
     (a → ¬b) ∧ (¬a ∧ ¬b) → ¬c

1. Let a be “The company has suffered a loss”.  
   Let b be “The economy is improving”.  
   Let c be “The employees lose their jobs”.
   1. If the company hasn’t suffered a loss and the economy is improving then the employees don’t lose their jobs.
   2. Let a be “7 + 4 < 11”.  
      Let b be “The earth is rotating around the sun”.  
      a → b  
      True, because since the antecedent is false (11 is not < 11), the statement is true.
   3. Let a be “-12 > 10”.  
      Let b be “7 \* 3 ≠ 22”.  
      a ∨ b  
      True, since at least one side of the disjunction is true, that side being: 7 \* 3 not being equal to 22, it is 21.
   4. Let a be “The earth has one moon”.  
      Let b be “The earth is flat”.  
      a → b  
      False, this is the only combination for an implication to be false. The precedent is true, the earth has one moon, and the consequent is false, since we know the earth is spherical.
   5. Let a be “2 + 5 > 23”.  
      Let b be “9 > 20”.  
      a → b  
      True, since the precedent of the implication is false.
2. (¬p → q) ↔ (q ∧ ¬r) Implication Relation  
   ≡ (¬¬p ∨ q) ↔ (q ∧ ¬r) Double Negation  
   ≡ (p ∨ q) ↔ (q ∧ ¬r) Bidirectional  
   ≡ ((p ∨ q) → (q ∧ ¬r)) ∧ ((q ∧ ¬r) → (p ∨ q)) Implication Relation  
   ≡ (¬(p ∨ q) ∨ (q ∧ ¬r)) ∧ (¬(q ∧ ¬r) ∨ (p ∨ q))  
     
     
   They are logically equivalent as shown in the truth tables. The final result is shown highlighted in blue.

p q r | ( ¬ ( p ∨ q ) ∨ ( q ∧ ¬ r ) ) ∧ ( ¬ ( q ∧ ¬ r ) ∨ ( p ∨ q ) )  
---------------------------------------------------------------------  
T T T | F T T T F T F F T F T T F F T T T T T

T T F | F T T T T T T T F T F T T T F T T T T  
T F T | F T T F F F F F T F T F F F T T T T F  
T F F | F T T F F F F T F F T F F T F T T T F  
F T T | F F T T F T F F T F T T F F T T F T T  
F T F | F F T T T T T T F T F T T T F T F T T  
F F T | T F F F T F F F T T T F F F T T F F F  
F F F | T F F F T F F T F T T F F T F T F F F

p q r | ( ¬ p → q ) ↔ ( q ∧ ¬ r )

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T T T | F T T T F T F F T

T T F | F T T T T T T T F  
T F T | F T T F F F F F T  
T F F | F T T F F F F T F  
F T T | T F T T F T F F T  
F T F | T F T T T T T T F  
F F T | T F F F T F F F T  
F F F | T F F F T F F T F

* 1. “Study Hard!” is not a proposition, since it is an exclamatory statement, and cannot be true or false.
  2. “Why do I have to study this stuff?” is not a proposition because it is an interrogative statement and cannot be true or false.
  3. “Life is like a box of chocolates.” Is a proposition because it declaring life is like a box of chocolates which can be true or false. If it were less general and said ‘your life’, it would no longer be a proposition because ‘your life’ depends on who is reading it and so it could be true and false.
  4. Contingency

x y | ( y ∧ x ) ↔ ( ¬ ( y ∨ x ) ∧ ¬ ( y ∧ ¬ x ) )  
-------------------------------------------------  
T T | T T T F F T T T F T T F F T  
T F | F F T T F F T T F T F F F T  
F T | T F F T F T T F F F T T T F  
F F | F F F F T F F F T T F F T F

* 1. Tautology

p q | ¬ ( ( ¬ p ∧ q ) ∨ ( ¬ q ∧ p ) ) ∨ ¬ ( ( ¬ p ∨ q ) ∧ ( ¬ q ∨ p ) )

-----------------------------------------------------------------------  
T T | T F T F T F F T F T T F F T T T T F T T T  
T F | F F T F F T T F T T T T F T F F F T F T T  
F T | F T F T T T F T F F T T T F T T F F T F F  
F F | T T F F F F T F F F T F T F T F T T F T F

* 1. Tautology

x y z | ( ¬ y ∧ ¬ z ) → ¬ ( ( y ∨ x ) ∧ ( ¬ x ∨ z ) )

-----------------------------------------------------  
T T T | F T F F T T F T T T T F T T T

T T F | F T F T F T T T T T F F T F F

T F T | T F F F T T F F T T T F T T T

T F F | T F T T F T T F T T F F T F F

F T T | F T F F T T F T T F T T F T T

F T F | F T F T F T F T T F T T F T F

F F T | T F F F T T T F F F F T F T T

F F F | T F T T F T T F F F F T F T F

* 1. ¬(¬(z → (a ∧ b)) → (¬a ∨ ¬b)) Implication Relation  
     ≡ ¬(¬¬(¬z ∨ (a ∧ b)) ∨ (¬a ∨ ¬b)) Double Negation  
     ≡ ¬((¬z ∨ (a ∧ b)) ∨ (¬a ∨ ¬b)) De Morgan’s Law  
     ≡ ¬(¬z ∨ (a ∧ b)) ∧ ¬(¬a ∨ ¬b) De Morgan’s Law  
     ≡ (z ∧ ¬(a ∧ b)) ∧ (a ∧ b) De Morgan’s Law  
     ≡ (z ∧ (¬a ∨ ¬b)) ∧ (a ∧ b) De Morgan’s Law  
     ≡ (z ∧ ¬(a ∧ b)) ∧ (a ∧ b) Associativity Law  
     ≡ z ∧ (¬(a ∧ b) ∧ (a ∧ b)) Contradiction   
     ≡ z ∧ F Domination Rules  
     ≡ F : Contradiction
  2. ((y ∨ z) ∧ (x ∨ z)) ∨ ((z → ¬x) ∧ (z → y)) Implication  
     ≡ ((y ∨ z) ∧ (x ∨ z)) ∨ ((¬z ∨ ¬x) ∧ (¬z ∨ y)) Distributive Law  
     ≡ (z ∨ (x ∧ y)) ∨ (¬z ∨ (¬x ∧ y)) Associativity Law  
     ≡ z ∨ (x ∧ y) ∨ ¬z ∨ (¬x ∧ y) Communitive Law  
     ≡ z ∨ ¬z ∨ (x ∧ y) ∨ (¬x ∧ y) Law of Excluded Middle  
     ≡ T ∨ (x ∧ y) ∨ (¬x ∧ y) Domination Rules  
     ≡ T ∨ (¬x ∧ y) Domination Rules  
     ≡ T : Tautology
  3. (¬y ∧ ¬z) → ¬((y ∨ x) ∧ (¬x ∨ z)) Implication Relation  
     ≡ ¬(¬y ∧ ¬z) ∨ ¬((y ∨ x) ∧ (¬x ∨ z)) De Morgan’s Law  
     ≡ (y ∨ z) ∨ (¬(y ∨ x) ∨ ¬(¬x ∨ z)) De Morgan’s Law  
     ≡ (y ∨ z) ∨ ((¬y ∧ ¬x) ∨ (x ∧ ¬z)) De Morgan’s Law  
     ≡ (y ∨ z) ∨ (¬(y ∨ x) ∨ (x ∧ ¬z)) Associativity Law  
     ≡ (y ∨ z) ∨ ¬(y ∨ x) ∨ (x ∧ ¬z)   
     ≡ T : Tautology

1. K(x) is “x knows how to drive”.  
    O(x) is “x owns a motorcycle”.  
    A(x) is “x has a ticket to the auto show”.  
    U is “all humans”.
   1. There exists a human that knows how to drive, owns a motorcycle and doesn’t have a ticket to the auto show.
   2. Anybody that doesn’t own a motorcycle will not know how to drive.
   3. Anybody that knows how to drive and owns a motorcycle has a ticket to the auto show.
   4. There is somebody that doesn’t own a motorcycle -> that doesn’t know how to drive or doesn’t have a ticket to the auto show.
2. H(x) is “x can ski”.  
    S(x) is “x can skate”.  
    W(x) is “x loves winter”.  
    P(x) is “x plays soccer”.  
    U is “all humans”.
   1. ¬∃x (H(x) ∧ P(x))
   2. ∃x (¬W(x) ∧ S(x))
   3. ¬∀x (W(x))
   4. ∃!x (S(x) ∧ H(x))
   5. Some wireless network connections have the same bandwidth.
   6. Everybody that goes to Carleton knows logic.